JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 29th JANUARY 2025)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

SECTION-A

1. If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real root, is the interval (α,β), and $X = \{x \in Z : \alpha < x < \beta\}$, then $\sum_{x \in X} x^2$ is equal to

(1) 2109 (2) 2129

(3) 2139 (4) 2119

Ans. (3)

Sol. $(a-5)^2 - 8(15-3a) < 0$

$a^2 + 14a + 25 - 120 < 0$

$a^2 + 14a - 95 < 0$

$(a+19)(a-5) < 0$

$a \in (-19, 5)$

$\therefore -19 < x < 5$

$\therefore \sum_{x \in X} x^2 = (1^2 + 2^2 + \ldots + 4^2) + (1^2 + 2^2 + \ldots + 18^2)$

$= \frac{4 \times 5 \times 9}{6} + \frac{18 \times 19 \times 37}{6}$

$= 30 + 2109$

$= 2139$

2. If $\sin x + \sin^2 x = 1,\ x \in \left(0, \frac{\pi}{2}\right)$, then $(\cos^{12}x + \tan^{12}x) + 3(\cos^{10}x + \tan^{10}x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$ is equal to

(1) 4 (2) 3

(3) 2 (4) 1

Ans. (3)

Sol. $\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = \cos^2 x \Rightarrow \tan x = \cos x$

$\therefore$ Given expression

$= 2\cos^{12}x + 6[\cos^{10}x + \cos^8 x] + 2\cos^6 x$

$= 2[\sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x]$

$= 2\sin^3 x[(\sin x + 1)^3]$

$= 2[\sin^2 x + \sin x]^3$

$= 2$

3. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma$; β, γ are integers, then the value of $|\beta - \gamma|$ equals

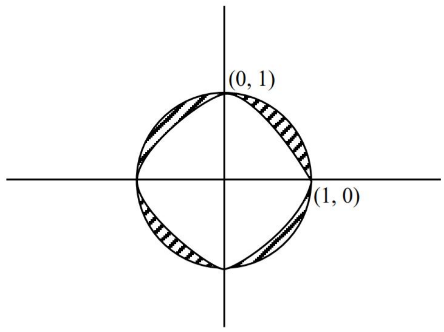
(1) 27 (2) 18

(3) 15 (4) 33

Ans. (4)

Sol. $C_1 : |y| = 1 - x^2$

$C_2 : x^2 + y^2 = 1$



$\therefore$ Required Area

$= \alpha = 4[\text{Area of circle in } 1^{st} \text{ quad.} - \int_0^1 (1-x^2)dx]$

$= 4\left[\frac{\pi}{4} - \left[x - \frac{x^3}{3}\right]_0^1\right]$

$\alpha = \pi - \frac{8}{3}$

$\therefore 3\alpha = 3\pi - 8$

$\therefore 9\alpha = 9\pi - 24$

$\therefore \beta = 9, \gamma = -24$

$\therefore |\beta - \gamma| = 33$

4. If the domain of the function $\log_5\left(18x - x^2 - 77\right)$ is (α,β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2 + 3x - 2}{x^2 - 3x - 4}\right)$ is (γ,δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 195 (2) 174

(3) 186 (4) 179

Ans. **(3)**

Sol. $f_1(x) = \log_5(18x - x^2 - 77)$

$\therefore 18x - x^2 - 77 > 0$

$x^2 - 18x + 77 < 0$

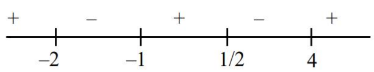
$x \in (7, 11)\ \ \alpha = 7, \beta = 11$

$f_2(x) = \log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$

$\therefore\ \ x-1>0,\ x-1\neq 1,\ \frac{2x^2+3x-2}{x^2-3x-4}>0$

$x>1,\ x\neq 2,\ \frac{(2x-1)(x+2)}{(x-4)(x+1)}>0$

$x>1,\ x\neq 2,$



$\therefore\ \ x \in (4, \infty)$

$\therefore\ \ \gamma = 4$

$\therefore\ \ \alpha^2+\beta^2+\gamma^2 = 49 + 121 + 16$

$= 186$

5. Let the function $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos|x|$ be not differentiable at the two points $x = \alpha = 2$ and $x = \beta$. Then the distance of the point (α,β) from the line $12x + 5y + 10 = 0$ is equal to :

(1) 3 (2) 4

(3) 2 (4) 5

Ans. **(1)**

Sol. $\cos|x|$ is always differentiable

$\therefore$ we have to check only for $|x^2 - ax + 2|$

$\therefore$ Not differentiable at

$x^2 - ax + 2 = 0$

One root is given, $\alpha = 2$

$\therefore\ \ 4 - 2a + 2 = 0$

$a = 3$

$\therefore$ other root $\beta = 1$

but for $x = 1$ f(x) is differentiable

(Drop)

6. Let a straight line L pass through the point P(2,–1,3) and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is :

(1) 2 (2) $\sqrt{10}$

(3) 3 (4) $2\sqrt{3}$

Ans. **(3)**

Sol. Vector parallel to 'L'

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 3 & 4\end{vmatrix} = 10\hat{i}-10\hat{j}+5\hat{k}$$

$= 5(2\hat{i}-2\hat{j}+\hat{k})$

Equation of 'L'

$\frac{x-2}{2}=\frac{y+1}{-2}=\frac{z-3}{1}=\lambda$ (say)

Let $Q(2\lambda + 2, -2\lambda - 1, \lambda + 3)$

$\Rightarrow 2\lambda + 2 = 0 \Rightarrow \lambda = -1$

$\Rightarrow Q\ (0, 1, 2)$

$d(P, Q) = 3$

7. Let $S = \mathbf{N} \cup \{0\}$. Define a relation **R** from S to **R** by :

$$\mathbf{R} = \left\{(x,y) : \log_e y = x\log_e\left(\frac{2}{5}\right), x\in S, y \in \mathbf{R}\right\}.$$

Then, the sum of all the elements in the range of **R** is equal to

(1) $\frac{3}{2}$ (2) $\frac{5}{3}$

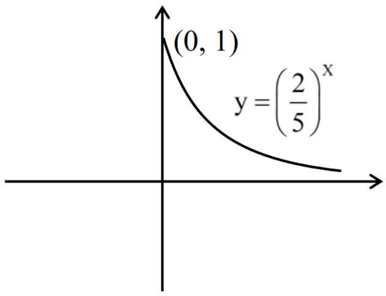
(3) $\frac{10}{9}$ (4) $\frac{5}{2}$

Ans. **(2)**

Sol. $S = \{0, 1, 2, 3 \ldots..\}$

$\log_e y = x\log_e\left(\frac{2}{5}\right)$

$\Rightarrow\ \ y = \left(\frac{2}{5}\right)^x$



Required

$$\text{Sum} = 1+\left(\frac{2}{5}\right)^{1}+\left(\frac{2}{5}\right)^{2}+\left(\frac{2}{5}\right)^{3}+.....-=\frac{1}{1-\frac{2}{5}}=\frac{5}{3}$$

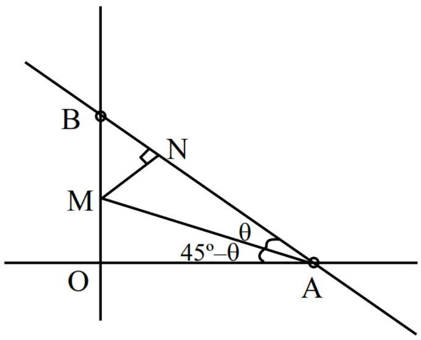
8. Let the line x + y = 1 meet the axes of x and y at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and AN : NB = λ : 1, then the sum of all possible value(s) of is λ :

(1) $\frac{1}{2}$ (2) $\frac{13}{6}$

(3) $\frac{5}{2}$ (4) 2

Ans. (4)

Sol.



Area of $\Delta AOB = \frac{1}{2}$

Area of $\Delta AMN = \frac{4}{9}\times\frac{1}{2}=\frac{2}{9}$

Equation of AB is x + y = 1

$OA = 1,\ AM = \sec(45° - \theta)$

$AN = \sec(45° - \theta)\cos\theta$

$MN = \sec(45° - \theta)\sin\theta$

$$Ar(\Delta AMN) = \frac{1}{2}\times \sec^2(45° - \theta)\sin\theta.\cos\theta=\frac{2}{9}$$

$\Rightarrow \tan\theta = 2, \frac{1}{2}$

$\tan\theta = 2$ is rejected

$$\frac{AN}{NB}=\frac{\lambda}{1}=\cot\theta=2$$

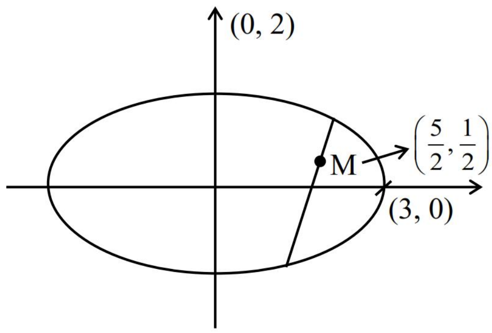
9. If αx + βy = 109 is the equation of the chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, whose mid point is $\left(\frac{5}{2},\frac{1}{2}\right)$, then α + β is equal to

(1) 37 (2) 46

(3) 58 (4) 72

Ans. (3)

Sol.



Equation of chord $T = S_1$

$$\frac{5}{2}\left(\frac{x}{9}\right)+\frac{1}{2}\left(\frac{y}{4}\right)=\frac{25}{36}+\frac{1}{16}$$

$$\Rightarrow \frac{5x}{18}+\frac{y}{8}=\frac{100+9}{144}=\frac{109}{144}$$

$\Rightarrow 40x + 18y = 109$

$\Rightarrow \alpha = 40, \beta = 18$

$\Rightarrow \alpha + \beta = 58$

10. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement, is :

(1) PRNAKU (2) PRKANU

(3) PRKAUN (4) PRNAUK

Ans. (3)

Sol. A, K, N, P, R, U

$\boxed{A}$ .... $= \underline{\lfloor 5} = 120$

$\boxed{K}$ .... $= \underline{\lfloor 5} = 120$

$\boxed{N}$ .... $= \underline{\lfloor 5} = 120$

$\boxed{P}\,\boxed{A}$ .... $= \underline{\lfloor 4} = 24$

$\boxed{P}\,\boxed{K}$ .... $= \underline{\lfloor 4} = 24$

$\boxed{P}\,\boxed{N}$ .... $= \underline{\lfloor 4} = 24$

$\boxed{P}\,\boxed{R}\,\boxed{A}$ .... $= \underline{\lfloor 3} = 6$

$\boxed{P}\,\boxed{R}\,\boxed{K}\,\boxed{A}\,\boxed{N}\,\boxed{U}$ $= 1$

$\boxed{P}\,\boxed{R}\,\boxed{K}\,\boxed{A}\,\boxed{U}\,\boxed{N}$ $\underline{\quad = 1}$

Total = 440

$\Rightarrow$ 440[th] word

11. Let α, β $(\alpha \neq \beta)$ be the values of m, for which the equations $x + y + z = 1$; $x + 2y + 4z = m$ and $x + 4y + 10z = m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^{\alpha} + n^{\beta}\right)$ is equal to :

(1) 440 (2) 3080

(3) 3410 (4) 560

Ans. **(1)**

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20 - 16) - 1(10 - 4) + 1(4 - 2)$

$= 4 - 6 + 2 = 0$

For infinite solutions

$\Delta_x = \Delta_y = \Delta_z = 0$

$m^2 - 3x + 2 = 0$

$m = 1, 2$

$\alpha = 1, \beta = 2$

$\therefore \sum_{n=1}^{10}(n^{\alpha} + n^{\beta}) = \sum_{n=1}^{10} n^1 + \sum_{n=1}^{10} n^2$

$= \frac{10(11)}{2} + \frac{10(11)(21)}{6}$

$= 55 + 385$

$= 440$

12. Let $A = [a_{ij}]$ be a matrix of order 3×3, with $a_{ij} = \left(\sqrt{2}\right)^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha + \beta\sqrt{2}$, $\alpha, \beta \in \mathbf{Z}$, then $\alpha + \beta$ is equal to

(1) 280 (2) 168

(3) 210 (4) 224

Ans. **(4)**

Sol. $A = \begin{bmatrix} \left(\sqrt{2}\right)^2 & \left(\sqrt{2}\right)^3 & \left(\sqrt{2}\right)^4 \\ \left(\sqrt{2}\right)^3 & \left(\sqrt{2}\right)^4 & \left(\sqrt{2}\right)^5 \\ \left(\sqrt{2}\right)^4 & \left(\sqrt{2}\right)^5 & \left(\sqrt{2}\right)^6 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$

$A^2 = 2^2 \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}$

$= 4\begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{bmatrix}$

Sum of elements of 3[rd] row $= 4(14 + 14\sqrt{2} + 28)$

$= 4(42 + 14\sqrt{2})$

$= 168 + 56\sqrt{2}$

$\alpha + \beta\sqrt{2}$

$\therefore\ \alpha + \beta = 168 + 56 = 224$

13. Let P be the foot of the perpendicular from the point (1, 2, 2) on the line L : $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$.

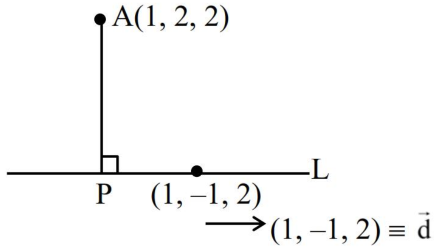
Let the line $\vec{r} = \left(-\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$, $\lambda \in \mathbf{R}$, intersect the line L at Q. Then $2(PQ)^2$ is equal to:

(1) 27 (2) 25

(3) 29 (4) 19

Ans. **(1)**

Sol.



$$L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2} = \mu$$

$P(\mu + 1, -\mu - 1, 2\mu + 2)$

$\overrightarrow{AP} \cdot \vec{d} = 0 \Rightarrow (\mu, -\mu - 3, 2\mu) \cdot (1, -1, 2) = 0$

$\Rightarrow \mu + \mu + 3 + 4\mu = 0 \Rightarrow \mu = -\frac{1}{2}$

$\therefore\ P\left(\frac{-1}{2} + 1, +\frac{1}{2} - 1, 2\left(\frac{-1}{2}\right) + 2\right)$

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$

Now general pt. on L_2 is $Q(-1 + \lambda, 1 - \lambda, -2 + \lambda)$

Equate it with general pt of L

$$\begin{array}{c|c|c} \mu+1=-1+\lambda & -\mu-1=1-\lambda & 2\mu+2=-2+\lambda \\ \mu=\lambda-2 & \mu=\lambda-2 & \downarrow \end{array}$$

$2(\lambda - 2) + 2 = -2 + \lambda$

$2\lambda - 4 + 2 = -2 + \lambda$

$\therefore\ \mu = -2\ ,\ \lambda = 0$

$\therefore\ Q \equiv (-1, 1 -2)$

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$ and $Q(-1, 1, -2)$

$$PQ = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{-1}{2}-1\right)^2 + (1+2)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{9}{4} + 9} = \sqrt{\frac{54}{4}}$$

$\therefore\ 2(PQ)^2 = 2\left(\frac{54}{4}\right) = 27$

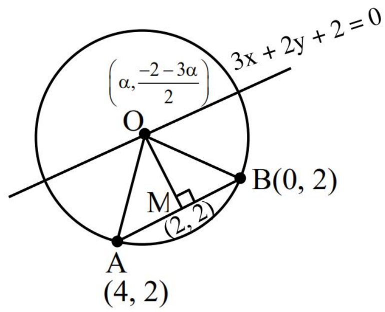
14. Let a circle C pass through the points (4, 2) and (0, 2), and its centre lie on $3x + 2y + 2 = 0$. Then the length of the chord, of the circle C, whose mid-point is (1, 2), is:

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Ans. (2)

Sol.



$M_{AB} = 0 \Rightarrow$ OM is vertical

$\Rightarrow\ \alpha = 2$

$\therefore$ Centre $(0) \equiv (2, -4)$

$r = OA = \sqrt{(2-4)^2 + (2+4)^2} = \sqrt{40}$

mid point of chord is $N \equiv (1, 2)$ $\therefore$ $ON = \sqrt{37}$

$\therefore$ length of chord $= 2\sqrt{r^2 - (ON)^2}$

$= 2\sqrt{40 - 37} = 2\sqrt{3}$

15. Let $A = [a_{ij}]$ be a 2×2 matrix such that $a_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is:

(1) $\frac{1}{4}$ (2) $\frac{3}{8}$

(3) $\frac{5}{8}$ (4) $\frac{3}{4}$

Ans. (2)

Sol. $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$= a_{11}a_{22} - a_{21}a_{12}$

$= \{-1, 0, 1\}$

x	P_i	P_iX_i	$P_iX_i^2$
–1	$\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$
0	$\frac{10}{16}$	0	0
1	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$
		$\sum P_iX_i = 0$	$\sum P_iX_i^2 = \frac{3}{8}$

$\therefore$ var(x) $= \sum P_iX_i^2 - (\sum P_iX_i)^2$

$= \frac{3}{8} - 0 = \frac{3}{8}$

16. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is 29/45, then n is equal to:

(1) 3 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. Bag 1 = {4W, 5B}

Bag 2 = {nW, 3B}

$$P\left(\frac{W}{Bag\,2}\right)=\frac{29}{45}$$

$$\Rightarrow P\left(\frac{W}{B_1}\right)\times P\left(\frac{W}{B_2}\right)+P\left(\frac{B}{B_1}\right)\times P\left(\frac{W}{B_2}\right)=\frac{29}{45}$$

$$\frac{4}{9}\times\frac{n+1}{n+4}+\frac{5}{9}\times\frac{n}{n+4}=\frac{29}{45}$$

$$\boxed{n=6}$$

17. The remainder, when 7^{103} is divided by 23, is equal to:

(1) 14 (2) 9

(3) 17 (4) 6

Ans. (1)

Sol. $7^{103} = 7\,(7^{102}) = 7\,(343)^{34} = 7\,(345-2)^{34}$

$7^{103} = 23K_1 + 7.2^{34}$

Now $7.2^{34} = 7\,.\,2^2\,.\,2^{32}$

$= 28\,.\,(256)^4$

$= 28\,(253+3)^4$

$\therefore 28\times 81 \Rightarrow (23+5)\,(69+12)$

$23K_2 + 60$

$\therefore$ Remainder = $\boxed{14}$

18. Let $f(x)=\int_0^x t\left(t^2-9t+20\right)dt$, $1 \le x \le 5$. If the range of f is $[\alpha, \beta]$, then $4(\alpha+\beta)$ equals:

(1) 157 (2) 253

(3) 125 (4) 154

Ans. (1)

Sol. $f'(x) = x^3 - 9x^2 + 20x = x(x-4)(x-5)$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ \hline & 0 & & 4 & & 5 & \end{array}$$

$$\therefore \mathbf{f(x)} = \frac{x^4}{4}-\frac{9x^3}{3}+\frac{20x^2}{2}$$

$$f(1) = \frac{1}{4} - 3 + 10 = \frac{29}{4} = \alpha$$

$$f(4) = \frac{256}{4} - 3(64)\,|+10(16) = 32 = \beta$$

$$4(\alpha+\beta)=4\left(\frac{29}{4}+32\right)=157$$

19. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-\hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i}+\hat{j}+\hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i}+\alpha\hat{j}+\hat{k}$, then the value of α is :

(1) $-\sqrt{3}$ (2) $\sqrt{6}$

(3) $-\sqrt{6}$ (4) $\sqrt{3}$

Ans. (3)

Sol. Let $\vec{v}=\hat{i}+\hat{j}+\hat{k}$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 2 & 3 & -1\end{vmatrix}$$

$$=-7\hat{i}+7\hat{j}+7\hat{k}$$

$$=-7(\hat{i}-\hat{j}-\hat{k})$$

Now $\hat{a}=\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ or $\frac{-\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

$\downarrow$ $\downarrow$

$$\cos\theta=\frac{\hat{a}.\vec{v}}{|\vec{v}|}=\frac{1-1-1}{\sqrt{3}\sqrt{3}}=\frac{-1}{3} \qquad \cos\theta=\frac{\hat{a}.\vec{v}}{|\vec{v}|}=\frac{-1+1+1}{3}=\frac{1}{3}$$

(rejected)

$$\Rightarrow \hat{a}=\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$$

Now $\cos\frac{\pi}{3} = \frac{\hat{a}\cdot(\hat{i}+\alpha\hat{j}+\hat{k})}{\sqrt{1+\alpha^2+1}}$

$\Rightarrow \frac{1}{2} = \frac{1-\alpha-1}{\sqrt{3}\sqrt{\alpha^2+2}}$

$\Rightarrow \frac{\sqrt{3}}{2}\sqrt{\alpha^2+2} = -\alpha \quad (\therefore\ \alpha < 0)$

$3\alpha^2 + 6 = 4\alpha^2$

$\Rightarrow \alpha = -\sqrt{6}$

20. If for the solution curve y = f(x) of the differential equation $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$,

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to:

(1) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$ (2) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$

(3) $\frac{5-\sqrt{3}}{2\sqrt{2}}$ (4) $\frac{4-\sqrt{2}}{14}$

Ans. (4)

Sol. If $e^{\int \tan x\,dx} = e^{\ell n(\sec x)} = \sec x$

$\therefore$ **y . sec x** $= \int \left\{\frac{2+\sec x}{(1+2\sec x)^2}\right\} \sec x\, dx$

$= \int \frac{2\cos x+1}{(\cos x+2)^2}dx$ Let $\cos x = \frac{1-t^2}{1+t^2}$

$= \int \frac{2\left(\frac{1-t^2}{1+t^2}\right)+1}{\left(\frac{1-t^2}{1+t^2}+2\right)^2} 2dt$

$= \int \frac{2-2t^2+1+t^2}{(1-t^2+2+2t^2)^2} \times 2dt$

$= 2\int \frac{3-t^2}{(t^2+3)^2}dt$

Let $t + \frac{3}{t} = u$

$\left(1-\frac{3}{t^2}\right)dt = du$

$= -2\int \frac{du}{u^2}$

$y\,.\,(\sec x) = \frac{2}{u} + c$

$\boxed{y.\sec x = \frac{2}{t+\frac{3}{t}} + c}$ ....(I)

At $x = \frac{\pi}{3}$, $t = \tan\frac{x}{2} = \frac{1}{\sqrt{3}}$

$2.\frac{\sqrt{3}}{10} = \frac{2}{\frac{1}{\sqrt{3}}+3\sqrt{3}} + c$

$2.\frac{\sqrt{3}}{10} = \frac{2\sqrt{3}}{10} + c \Rightarrow C = 0$

At $x = \frac{\pi}{4}$, $t = \tan\frac{x}{2} = \sqrt{2} - 1$

$\therefore\ y.\sqrt{2} = \frac{2}{\sqrt{2}-1+\frac{3}{\sqrt{2}-1}}$

$y.\sqrt{2} = \frac{2(\sqrt{2}-1)}{6-2\sqrt{2}}$

$y = \frac{\sqrt{2}(\sqrt{2}-1)}{2(3-\sqrt{2})} = \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}-1}{7}$

$= \frac{4-\sqrt{2}}{14}$

SECTION-B

21. If $24\int_0^{\frac{\pi}{4}}\left(\sin\left|4x-\frac{\pi}{12}\right| + [2\sin x]\right)dx = 2\pi + \alpha$, where [·] denotes the greatest integer function, then α is equal to ________.

Ans. (12)

Sol. $= 24\int_0^{\frac{\pi}{48}} -\sin\left(4x-\frac{\pi}{12}\right) + \int_{\pi/48}^{\pi/4}\sin\left(4x-\frac{\pi}{12}\right)$

$+\int_0^{\frac{\pi}{6}}[0]dx + \int_{\pi/6}^{\pi/4}[\underset{1}{\underset{\downarrow}{2\sin x}}]dx$

$= 24\left[\frac{\left(1-\cos\frac{\pi}{12}\right)}{4} - \frac{\left(-\cos\frac{\pi}{12}-1\right)}{4}\right] + \frac{\pi}{4} - \frac{\pi}{6}$

$= 24\left(\frac{1}{2}+\frac{\pi}{12}\right) = 2\pi + 12$

$\alpha = 12$

22. If $\lim_{t\to 0}\left(\int_0^1 (3x+5)^t\,dx\right)^{\frac{1}{t}} = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then α is equal to ________.

Ans. (64)

Sol. 1^∞ form

$$\text{Now } L = e^{\lim_{t\to 0}\frac{1}{t}\left(\left.\frac{(3x+5)^{t+1}}{3(t+1)}\right|_0^1 - 1\right)}$$

$$= e^{\lim_{t\to 0}\frac{8^{t+1}-5^{t+1}-3t-3}{3t(t+1)}}$$

$$= e^{\frac{8\ell n 8 - 5\ell n 5 - 3}{3}}$$

$$= \left(\frac{8}{5}\right)^{2/3}\left(\frac{64}{5}\right) = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{2/3}$$

On comparing

$\alpha = 64$

23. Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + \ldots + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \ldots + a_{2024}$ is equal to ________.

Ans. (11132)

Sol. $a_1 + a_5 + a_{10} + \ldots + a_{2020} + a_{2024} = 2233$

In an A.P. the sum of terms equidistant from ends is equal.

$a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} \ldots$

$\Rightarrow 203$ pairs

$\Rightarrow 203(a_1 + a_{2024}) = 2233$

Hence,

$$S_{2024} = \frac{2024}{2}(a_1 + a_{2024})$$

$= 1012 \times 11$

$= 11132$

24. Let integers $a, b \in [-3, 3]$ be such that $a + b \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right| = 1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$, $z \in C$, where ω and ω^2 are the roots of $x^2 + x + 1 = 0$, is equal to ________.

Ans. (10)

Sol. $a, b \in I, -3 \le a, b \le 3, a + b \neq 0$

$|z - a| = |z + b|$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix} = 1$$

$\Rightarrow z^3 = 1$

$\Rightarrow z = \omega, \omega^2, 1$

Now

$|1-a| = |1+b|$

$\Rightarrow 10$ pairs

25. Let $y^2 = 12x$ the parabola and S be its focus. Let PQ be a focal chord of the parabola such that (SP) (SQ) $= \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}\,y = \beta$, then $\beta - \alpha$ is equal to ________.

Ans. (1328)

Sol. $y^2 = 12x \quad a = 3 \quad SP \times SQ = \frac{147}{4}$

Let $P(3t^2, 6t)$ and $t_1t_2 = -1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

$S(3, 0)$

$SP \times SQ = PM_1 \times QM_2$

(dist. from directrix)

$$= (3 + 3t^2)\left(3+\frac{3}{t^2}\right) = \frac{147}{4}$$

$$\Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12}$$

$$t^2 = \frac{3}{4}, \frac{4}{3}$$

$$t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$$

considering $t = \frac{-\sqrt{3}}{2}$

$P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q\left(4, 4\sqrt{3}\right)$

Hence, diametric circle:

$$(x-4)\left(x-\frac{9}{4}\right) + (y+3\sqrt{3})(y-4\sqrt{3}) = 0$$

$$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$$

$\Rightarrow \alpha = 400$, $\beta = 1728$

$\beta - \alpha = 1328$

PHYSICS

SECTION-A

26. The difference of temperature in a material can convert heat energy into electrical energy. To harvest the heat energy, the material should have

(1) low thermal conductivity and low electrical conductivity

(2) high thermal conductivity and high electrical conductivity

(3) low thermal conductivity and high electrical conductivity

(4) high thermal conductivity and low electrical conductivity

Ans. (3)

Sol. See-back effect

Low thermal conductivity

High electrical conductivity

Ans. (3)

27. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

Reason (R) : In isothermal process, PV = constant, while in adiabatic process PV^{γ} = constant. Here γ is the ratio of specific heats, P is the pressure and V is the volume of the ideal gas.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

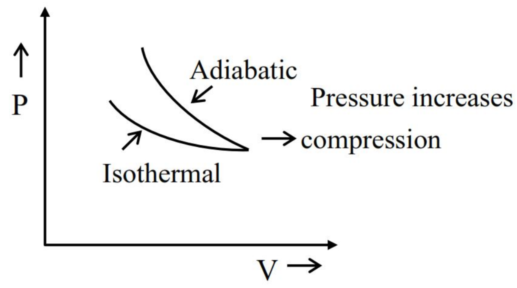
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

(4) **(A)** is false but **(R)** is true

Ans. (3)

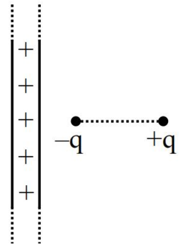
Sol.



$$\left(\frac{dP}{dV}\right)_{\text{Adiabatic}} > \left(\frac{dP}{dV}\right)_{\text{Isothermal}}$$

Ans. (3)

28. An electric dipole is placed at a distance of 2 cm from an infinite plane sheet having positive charge density σ_0. Choose the correct option from the following.



(1) Torque on dipole is zero and net force is directed away from the sheet.

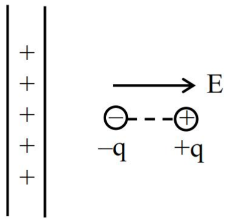
(2) Torque on dipole is zero and net force acts towards the sheet.

(3) Potential energy of dipole is minimum and torque is zero.

(4) Potential energy and torque both are maximum

Ans. (3)

Sol.



Here $E = \frac{\sigma}{2\epsilon_0}, \vec{\tau} = \vec{P} \times \vec{E}$

$\vec{\tau} = 0$

$U = -\vec{P} \cdot \vec{E} \quad U \rightarrow$ minimum

Ans. (3)

29. In an experiment with photoelectric effect, the stopping potential.

(1) increases with increase in the wavelength of the incident light

(2) increases with increase in the intensity of the incident light

(3) is $\left(\frac{1}{e}\right)$ times the maximum kinetic energy of the emitted photoelectrons

(4) decreases with increase in the intensity of the incident light

Ans. (3)

Sol. $\frac{hC}{\lambda} = W + eV_S$

$\frac{hC}{\lambda} = W + (K_{max})$

$\therefore V_S = \frac{K_{max}}{e}$

Ans. (3)

30. A point charge causes an electric flux of -2×10^4 Nm^2C^{-1} to pass through a spherical Gaussian surface of 8.0 cm radius, centred on the charge. The value of the point charge is :

(Given $\in_0 = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$)

(1) -17.7×10^{-8}C (2) -15.7×10^{-8}C

(3) 17.7×10^{-8}C (4) 15.7×10^{-8}C

Ans. (1)

Sol. $\phi = -2\times10^4 \frac{Nm^2}{C}$

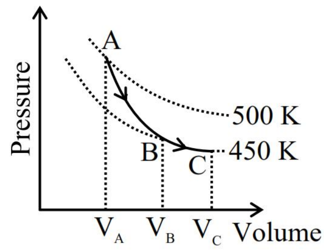
r = 8.0 cm

$\phi = \frac{q}{\in_0} \Rightarrow q = \in_0 \phi$

$= (8.85 \times 10^{-12}) \times (-2 \times 10^4)$

$q = -17.7 \times 10^{-8}$ C

Ans. (1)

31. 

A poly-atomic molecule ($C_V = 3R$, $C_P = 4R$, where R is gas constant) goes from phase space point A($P_A = 10^5$ Pa, $V_A = 4 \times 10^{-6}m^3$) to point B ($P_B = 5 \times 10^4$ Pa, $V_B = 6 \times 10^{-6}m^3$) to point C ($P_C = 10^4$ Pa, $V_C = 8 \times 10^{-6}m^3$). A to B is an adiabatic path and B to C is an isothermal path.
The net heat absorbed per unit mole by the system is :

(1) 500R(ln3 + ln4) (2) 450R(ln4 – ln3)

(3) 500R ln2 (4) 400R ln4

Ans. (2)

Sol. $\Delta Q_{AB} = 0$ adiabatic

$\Delta Q_{BC} = \Delta W_{BC}$

$= nRT\ell n\left(\frac{V_C}{V_B}\right) = 450R\ell n\left(\frac{8\times10^{-6}}{6\times10^{-6}}\right)$

$= 450R\ell n\left(\frac{4}{3}\right) = 450R\ (\ell n4 - \ell n3)$

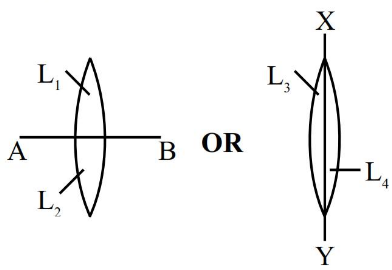
$\therefore \Delta Q = \Delta Q_{AB} + \Delta Q_{BC}$

$\Delta Q = 450\ R\ (\ell n4 - \ell n3)$

Note : Solution is based on direct data. B and C are not satisfying the condition of isothermal process.

Ans. (2)

32. Two identical symmetric double convex lenses of focal length *f* are cut into two equal parts L_1, L_2 by AB plane and L_3, L_4 by XY plane as shown in figure respectively. The ratio of focal lengths of lenses L_1 and L_3 is



(1) 1 : 4 (2) 1 : 1

(3) 2 : 1 (4) 1 : 2

Ans. (4)

Sol. $f_{L_1} = f_{L_2} = f$

$f_{L_3} = f_{L_4} = 2f$

$\therefore f_{L_1} : f_{L_3} = 1:2$

Ans. (4)

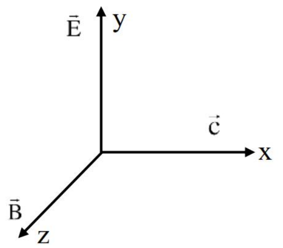
33. A plane electromagnetic wave propagates along the + x direction in free space. The components of the electric field, $\vec{E}$ and magnetic field, $\vec{B}$ vectors associated with the wave in Cartesian frame are :

(1) E_y, B_x (2) E_y, B_z

(3) E_x, B_y (4) E_z, B_y

Ans. (2)

Sol.

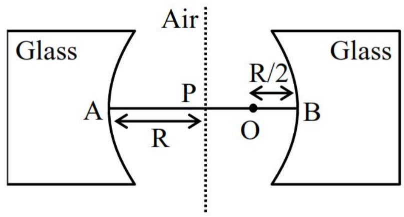


Direction of propogation

$= \vec{E} \times \vec{B}$

Ans. (2)

34.



Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object O is placed midway, between P and B. The separation between the images of O, formed by each refracting surface is :

(1) 0.214R

(2) 0.114R

(3) 0.411R

(4) 0.124R

Ans. (2)

Sol. For B

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{V} + \frac{1}{\frac{R}{2}} = \frac{0.5}{-R}$$

$$\frac{1.5}{V} = -\frac{1}{2R} - \frac{2}{R}$$

$$\frac{1.5}{V} = \frac{-5}{2R} \Rightarrow V_B = -0.6R$$

For A

$$\frac{1.5}{V} + \frac{2}{3R} = \frac{0.5}{-R}$$

$$\frac{1.5}{V} = -\frac{1}{2R} - \frac{2}{3R}$$

$$\frac{1.5}{V} = -\frac{7}{6R}$$

$$V_A = -\frac{9}{7}R$$

Distance between images

$$= 2R - \left(0.6R + \frac{9}{7}R\right) = 0.114\ R$$

option (2)

35. Two bodies A and B of equal mass are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the bodies oscillate vertically such that their amplitudes are equal, the ratio of the maximum velocity of A to the maximum velocity of B is

(1) $\sqrt{\frac{k_1}{k_2}}$ (2) $\frac{k_1}{k_2}$

(3) $\frac{k_2}{k_1}$ (4) $\sqrt{\frac{k_2}{k_1}}$

Ans. (1)

Sol. $V_1 = A_1\omega_1$

$V_2 = A_2\omega_2$

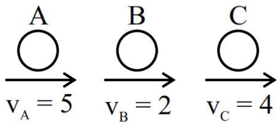
$A_1 = A_2$

$$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \frac{\sqrt{\frac{K_1}{m}}}{\sqrt{\frac{K_2}{m}}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{K_1}{K_2}}$$

36. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) :



Three identical spheres of same mass undergo one dimensional motion as shown in figure with initial velocities $v_A = 5$ m/s, $v_B = 2$ m/s, $v_C = 4$ m/s If we wait sufficiently long for elastic collision to happen, then $v_A = 4$ m/s, $v_B = 2$ m/s, $v_C = 5$ m/s will be the final velocities.

Reason (R) : In an elastic collision between identical masses, two objects exchange their velocities.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

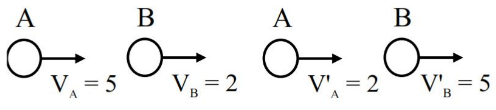
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

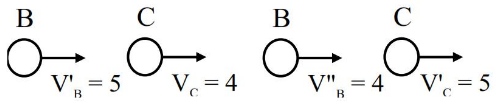
(4) **(A)** is false but **(R)** is true

Ans. (4)

Sol. In elastic collision for same mass, velocities interchange



Before collision **After collision**



Before collision **After collision**

$V'_A = 2\text{m/s} \quad V''_B = 4\text{m/s} \quad V'_C = 5\text{m/s}$

Ans. (4)

37. A sand dropper drops sand of mass m(t) on a conveyer belt at a rate proportional to the square root of speed (v) of the belt, i.e. $\frac{dm}{dt} \propto \sqrt{v}$. If P is the power delivered to run the belt at constant speed then which of the following relationship is true ?

(1) $P^2 \propto v^3$ (2) $P \propto \sqrt{v}$

(3) $P \propto v$ (4) $P^2 \propto v^5$

Ans. (4)

Sol. Power $= \vec{F} \cdot \vec{V}$

$$F = \frac{dp}{dt}[p = mv]$$

$$F = \left(\frac{dm}{dt}\right)v = C\left(\sqrt{v}\right)v$$

$$F = Cv^{\frac{3}{2}}$$

Power $= C(v^{3/2})v = Cv^{5/2}$

$p^2 \propto v^5$

Ans. (4)

38. A convex lens mode of glass (refractive index = 1.5) has focal length 24 cm in air. When it is totally immersed in water (refractive index = 1.33), its focal length changes to

(1) 72 cm (2) 96 cm

(3) 24 cm (4) 48 cm

Ans. (2)

Sol. $$\frac{1}{8} = \left(\frac{\mu_\ell}{\mu_S} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{24} = (1.5 - 1)\left[\frac{2}{R}\right] \qquad \ldots \text{(i)}$$

$$\frac{1}{f'} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{2}{R}\right)$$

$$\frac{1}{f'} = \left(\frac{1.5 \times 3}{4} - 1\right)\frac{2}{R} \qquad \ldots \text{(ii)}$$

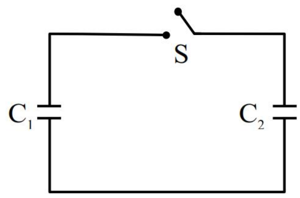
(i) divided by (ii)

$$\frac{f'}{24} = 4$$

f' = 96 cm

Ans. (2)

39. A capacitor, $C_1 = 6$ F is charged to a potential difference of $V_0 = 5V$ using a 5V battery. The battery is removed and another capacitor, $C_2 = 12$ μF is inserted in place of the battery. When the switch 'S' is closed, the charge flows between the capacitors for some time until equilibrium condition is reached. What are the charges (q_1 and q_2) on the capacitors C_1 and C_2 when equilibrium condition is reached.



(1) $q_1 = 15\ \mu C,\ q_2 = 30\ \mu C$

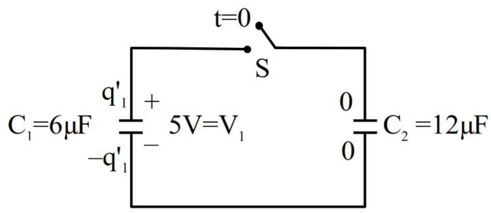
(2) $q_1 = 30\ \mu C,\ q_2 = 15\ \mu C$

(3) $q_1 = 10\ \mu C,\ q_2 = 20\ \mu C$

(4) $q_1 = 20\ \mu C,\ q_2 = 10\ \mu C$

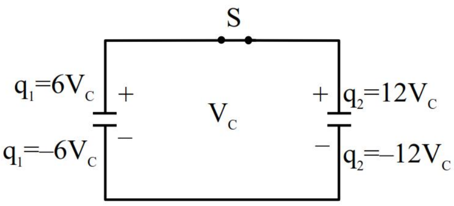
Ans. (3)

Sol.



$q'_1 = 6 \times 5 = 30\mu C$

Finally



$6V_C + 12V_C = 30 + 0$

$18V_C = 30$

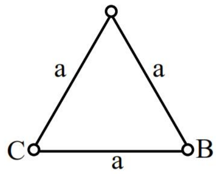
$$V_C = \frac{30}{18} = \frac{5}{3}\text{Volt}$$

$$\Rightarrow q_1 = \frac{6\times 5}{3} = 10\mu C$$

$$\Rightarrow q_2 = \frac{12\times 5}{3} = 20\mu C$$

Ans. (3)

40.



Three equal masses m are kept at vertices (A, B, C) of an equilateral triangle of side a in free space. At t = 0, they are given an initial velocity $\vec{V}_A = V_0\overrightarrow{AC}$, $\vec{V}_B = V_0\overrightarrow{BA}$ and $\vec{V}_C = V_0\overrightarrow{CB}$. Here, $\overrightarrow{AC}, \overrightarrow{CB}$ and $\overrightarrow{BA}$ are unit vectors along the edges of the triangle. If the three masses interact gravitationally, then the magnitude of the net angular momentum of the system at the point of collision is :

(1) $\frac{1}{2}amV_0$

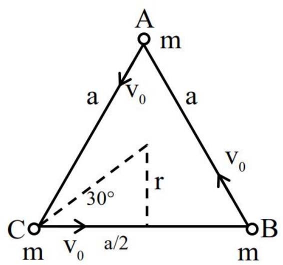
(2) $3\ a\ m\ V_0$

(3) $\frac{\sqrt{3}}{2}amV_0$

(4) $\frac{3}{2}amV_0$

Ans. (3)

Sol.



$$\tan 30° = \frac{2r}{a} = \frac{1}{\sqrt{3}}$$

$$r = \frac{a}{2\sqrt{3}}$$

$$L = (mvr_\perp)\times 3$$

$$= mv_0\frac{a}{2\sqrt{3}}\times 3$$

$$= \frac{\sqrt{3}}{2}mv_0a$$

41. Match **List-I** with **List-II**.

	List-I		List-II
(A)	Young's Modulus	(I)	$ML^{-1}T^{-1}$
(B)	Torque	(II)	$ML^{-1}T^{-2}$
(C)	Coefficient of Viscosity	(III)	$M^{-1}L^{3}T^{-2}$
(D)	Gravitational Constant	(IV)	$ML^{2}T^{-2}$

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Ans. **(4)**

Sol. (A) $[Y]=\dfrac{F}{A\left(\dfrac{\Delta\ell}{\ell}\right)} \Rightarrow \dfrac{MLT^{-2}}{L^2}=ML^{-1}T^{-2}$ (II)

(B) Torque $(\vec{\tau})=\vec{r}\times\vec{F}$

$(\vec{\tau})=L\times MLT^{-2}=ML^2T^{-2}$ (IV)

(C) Coefficient of viscosity $\Rightarrow F=\eta A\dfrac{dV}{dt}$

$\eta \rightarrow Pa{\cdot}sec$

$[\eta]=\dfrac{MLT^{-2}}{L^2}\times T=ML^{-1}T^{-1}$ (I)

(D) Gravitational constant (G)

$F=\dfrac{GM_1M_2}{r^2}$

$[G]=\dfrac{F\cdot r^2}{m_1m_2}=\dfrac{MLT^{-2}\times L^2}{M^2}=M^{-1}L^3T^{-2}$ (III)

42. Match **List-I** with **List-II**.

	List-I		List-II
(A)	Magnetic induction	(I)	Ampere meter^2
(B)	Magnetic intensity	(II)	Weber
(C)	Magnetic flux	(III)	Gauss
(D)	Magnetic moment	(IV)	Ampere meter

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Ans. **(2)**

Sol. (A) Magnetic induction $\rightarrow$ Gauss (III)

(B) Magnetic intensity

$\left(H=\dfrac{B}{\mu}\right)\rightarrow$ Ampere / meter (IV)

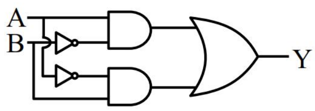
(C) Magnetic flux $\rightarrow$ Weber (Wb) (II)

(D) Magnetic moment $\rightarrow$ Ampere-meter^2

$(\vec{M}=i\vec{A})$

Note : None of the option(s) are correct but if we need to choose most appropriate option then the answer is (2)

43. The truth table for the circuit given below is :



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
1	0	0
1	1	0
0	1	1

(3)

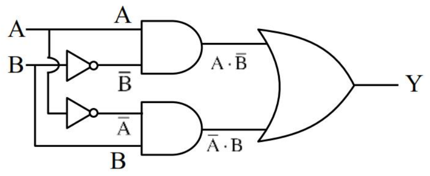
A	B	Y
0	0	0
1	0	1
0	1	0
1	1	0

(4)

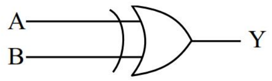
A	B	Y
0	0	0
1	1	1
1	0	1
0	1	1

Ans. **(1)**

Sol.



$Y = A\cdot\overline{B} + \overline{A}\cdot B$



XOR (Exclusive OR)

44. A cup of coffee cools from 90°C to 80°C in t minutes when the room temperature is 20°C. The time taken by the similar cup of coffee to cool from 80°C to 60°C at the same room temperature is :

(1) $\frac{13}{5}t$ (2) $\frac{10}{13}t$

(3) $\frac{13}{10}t$ (4) $\frac{5}{13}t$

Ans. (1)

Sol. By using average form of Newton's law of cooling

$$\frac{90-80}{t} = k\left(\frac{90+80}{2} - 20\right) \quad \dots (i)$$

$$\frac{80-60}{t'} = k\left(\frac{80+60}{2} - 20\right) \quad \dots (ii)$$

(i)/(ii)

$$\frac{10\times t'}{t\times 20} = \frac{65}{50}$$

$$t' = \frac{65}{50}\times 2t = \frac{65}{25}t = \frac{13}{5}t$$

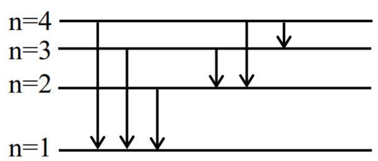
45. The number of spectral lines emitted by atomic hydrogen that is in the 4^{th} energy level, is

(1) 6 (2) 0

(3) 3 (4) 1

Ans. (1)

Sol.



Total possible transition = 6

SECTION-B

46. The magnetic field inside a 200 turns solenoid of radius 10 cm is 2.9×10^{-4} Tesla. If the solenoid carries a current of 0.29 A, then the length of the solenoid is __________ π cm.

Ans. (8)

Sol. Assuming long solenoid

$$B = \mu_0\left(\frac{N}{\ell}\right)i$$

$$\ell = \frac{\mu_0 N i}{B} = \frac{(4\pi\times10^{-7})(200)(0.29)}{2.9\times10^{-4}}\,m$$

$= 8\pi$ cm

47. A parallel plate capacitor consisting of two circular plates of radius 10 cm is being charged by a constant current of 0.15 A. If the rate of change of potential difference between the plates is 7×10^8 V/s then the integer value of the distance between the parallel plates is –

$\left(\text{Take}, \epsilon_0 = 9\times10^{-12}\frac{F}{m}, \pi = \frac{22}{7}\right)$ __________ μm.

Ans. (1320)

Sol. $$V = \frac{Q}{C} = \frac{it}{\left(\frac{\epsilon_0 A}{d}\right)} = \frac{itd}{\epsilon_0(\pi r^2)}$$

$$\Rightarrow d = \frac{\epsilon_0(\pi r^2)}{i}\left(\frac{v}{t}\right)$$

$$= \frac{(9\times10^{-12})\left(\frac{22}{7}\right)(0.1)^2}{0.15}(7\times10^8)\,m$$

$d = 1320$ μm

48. A physical quantity Q is related to four observables a, b, c, d as follows : $Q = \frac{ab^4}{cd}$

where, a = (60 ± 3)Pa; b = (20 ± 0.1)m;
c = (40 ± 0.2) Nsm^{-2} and d = (50 ± 0.1)m,

then the percentage error in Q is $\frac{x}{1000}$,

where $x =$ ______.

Ans. (77)

Sol. $$Q = \frac{ab^4}{cd}$$

$$\Rightarrow \frac{\Delta Q}{Q}\times100 = \left[\frac{\Delta a}{a} + 4\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}\right]\times100$$

$$\Rightarrow \frac{x}{1000} = \left[\frac{3}{60} + 4\left(\frac{0.1}{20}\right) + \left(\frac{0.2}{40}\right) + \frac{0.1}{50}\right]\times100$$

$\Rightarrow x = 7700$

49. Two planets, A and B are orbiting a common star in circular orbits of radii R_A and R_B, respectively, with $R_B = 2R_A$. The planet B is $4\sqrt{2}$ times more massive than planet A. The ratio $\left(\frac{L_B}{L_A}\right)$ of angular momentum (L_B) of planet B to that of planet A(L_A) is closest to integer __________.

Ans. **(8)**

Sol. $L = mv_0R = m\sqrt{\frac{GM}{R}}R = m\sqrt{GMR}$

here M is mass of star

$$\frac{L_B}{L_A} = \frac{m_B}{m_A}\sqrt{\frac{R_B}{R_A}}$$

$$= 4\sqrt{2}\sqrt{\frac{2}{1}}$$

$$\frac{L_B}{L_A} = 8$$

50. Two cars P and Q are moving on a road in the same direction. Acceleration of car P increases linearly with time whereas car Q moves with a constant acceleration. Both cars cross each other at time t = 0, for the first time. The maximum possible number of crossing(s) (including the crossing at t = 0) is ________ .

Ans. **(3)**

Sol. $a_P = kt$, k is constant

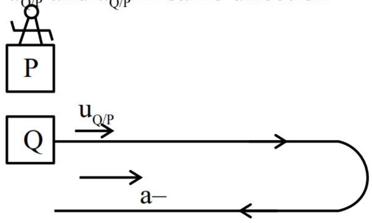
$a_Q = a$, a is constant

$a_{Q/P} = a_Q - a_P = a - kt$

as initial velocities are not mentioned in question, so will have to assume two cases.

Case-I

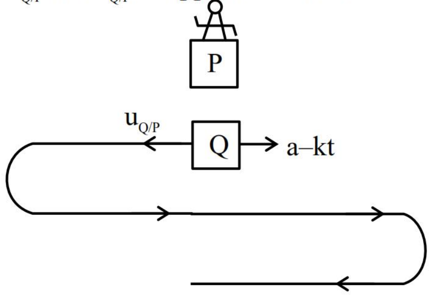
$u_{Q/P}$ and $a_{Q/P}$ in same direction



Total number of crossing = 2

Case-II

$u_{Q/P}$ and $a_{Q/P}$ in opposite direction



Total number of crossing = 3

CHEMISTRY

SECTION-A

51. The calculated spin-only magnetic moments of $K_3[Fe(OH)_6]$ and $K_4[Fe(OH)_6]$ respectively are :

(1) 4.90 and 4.90 B.M.

(2) 5.92 and 4.90 B.M.

(3) 3.87 and 4.90 B.M.

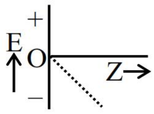
(4) 4.90 and 5.92 B.M.

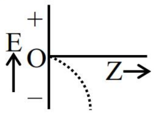
Ans. (2)

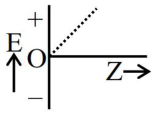
52. For hydrogen like species, which of the following graphs provides the most appropriate representation of E vs Z plot for a constant n ?

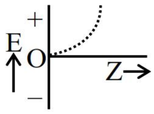
[E : Energy of the stationary state,

Z : atomic number, n = principal quantum number]

(1) 

(2) 

(3) 

(4) 

Ans. (2)

53. Given below are two statements :

Statement (I) : In partition chromatography, stationary phase is thin film of liquid present in the inert support.

Statement (II) : In paper chromatography, the material of paper acts as a stationary phase.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

(4) **Statement I** is false but **Statement II** is true

Ans. (2)

Sol. **Statement I is true.**

In partition chromatography, stationary phase is thin liquid film present in the inert support.

Statement II is false.

Because stationary phase in paper chromatography is water.

54. Identify the essential amino acids from below :

(A) Valine (B) Proline

(C) Lysine (D) Threonine

(E) Tyrosine

Choose the **correct** answer from the options given below :

(1) (A),(C) and (D) only

(2) (A),(C) and (E) only

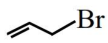
(3) (B),(C) and (E) only

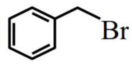
(4) (C),(D) and (E) only

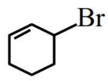
Ans. (1)

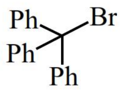
Sol. Valine, Lysine and Threonine are essential amino acids.

55. Which among the following halides will generate the most stable carbocation in Nucleophillic substitution reaction ?

(1) 

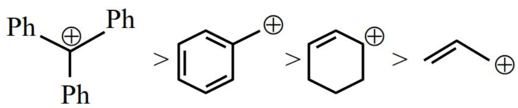
(2) 

(3) 

(4) 

Ans. (4)

Sol. Stability order of carbocation



56. Consider the equilibrium

$CO(g) + 3H_2(g) \rightleftharpoons CH_4(g) + H_2O(g)$

If the pressure applied over the system increases by two fold at constant temperature then

(A) Concentration of reactants and products increases.

(B) Equilibrium will shift in forward direction.

(C) Equilibrium constant increases since concentration of products increases.

(D) Equilibrium constant remains unchanged as concentration of reactants and products remain same.

Choose the **correct** answer from the options given below :

(1) (A) and (B) only (2) (A), (B) and (D) only

(3) (B) and (C) only (4) (A), (B) and (C) only

Ans. (1)

57. Given below are two statements :

Statement (I) : NaCl is added to the ice at 0°C, present in the ice cream box to prevent the melting of ice cream.

Statement (II) : On addition of NaCl to ice at 0°C, there is a depression in freezing point.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Ans. (2)

58. Given below are two statements :

Statement (I) : On nitration of m-xylene with HNO_3, H_2SO_4 followed by oxidation, 4-nitrobenzene-1, 3-dicarboxylic acid is obtained as the major product.

Statement (II) : CH_3 group is o/p-directing while-NO_2 group is m-directing group.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false

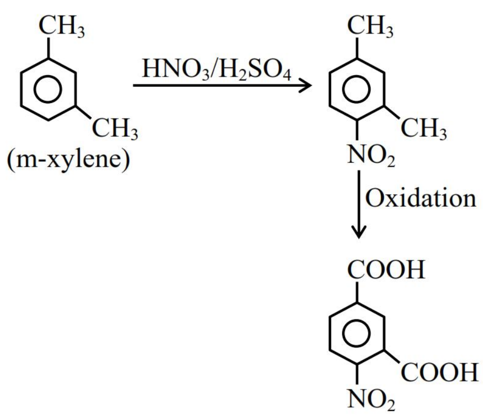
(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are true

(4) **Statement I** is true but **Statement II** is false

Ans. (3)

Sol. Statement-I



Statement-II

$-CH_3$ group is o/p directing while $-NO_2$ group is meta directing.

59. 0.1 M solution of KI reacts with excess of H_2SO_4 and KIO_3 solution. According to equation

$$5I^- + IO_3^- + 6H^+ \rightarrow 3I_2 + 3H_2O$$

Identify the **correct** statements :

(A) 200 mL of KI solution reacts with 0.004 mol of KIO_3

(B) 200 mL of KI solution reacts with 0.006 mol of H_2SO_4

(3) 0.5 L of KI solution produced 0.005 mol of I_2

(4) Equivalent weight of KIO_3 is equal to $\left(\frac{\text{Molecular weight}}{5}\right)$

Choose the **correct** answer from the options given below:

(1) (A) and (D) only
(2) (B) and (C) only
(3) (A) and (B) only
(4) (C) and (D) only

Ans. (1)

60. Match **List-I** with **List-II** :

List-I Applications		**List-II Batteries/Cell**	
(A)	Transistors	(I)	Anode - Zn/Hg ; Cathode - HgO + C
(B)	Hearing aids	(II)	Hydrogen fuel cell
(C)	Invertors	(III)	Anode – Zn; Cathode - Carbon
(D)	Apollo space ship	(IV)	Anode – Pb ; Cathode – Pb | PbO_2

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Ans. (1)

61. O_2 gas will be evolved as a product of electrolysis of :

(A) an aqueous solution of $AgNO_3$ using silver electrodes.

(B) an aqueous solution of $AgNO_3$ using platinum electrodes.

(C) a dilute solution of H_2SO_4 using platinum electrodes.

(D) a high concentration solution of H_2SO_4 using platinum electrodes.

Choose the **correct** answer from the options given below :

(1) (B) and (C) only
(2) (A) and (D) only
(3) (B) and (D) only
(4) (A) and (C) only

Ans. (1)

62. Identify the homoleptic complexes with odd number of d electrons in the central metal.

(A) $[FeO_4]^{2-}$ (B) $[Fe(CN)_6]^{3-}$

(C) $[Fe(CN)_5NO]^{2-}$ (D) $[CoCl_4]^{2-}$

(E) $[Co(H_2O)_3F_3]$

Choose the **correct** answer from the options given below :

(1) (B) and (D) only (2) (C) and (E) only

(3) (A), (B) and (D) only (4) (A), (C) and (E) only

Ans. (1)

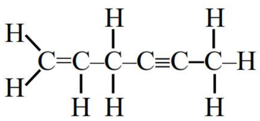
63. Total number of sigma (σ) ______and pi(π)______ bonds respectively present in hex-1-en-4-yne are :

(1) 13 and 3 (2) 11 and 3

(3) 3 and 13 (4) 14 and 3

Ans. (1)

Sol.



σ bonds = 13

π bonds = 3

64. If $C(diamond) \rightarrow C(graphite) + X\ kJ\ mol^{-1}$

$C(diamond) + O_2(g) \rightarrow CO_2(g) + Y\ kJ\ mol^{-1}$

$C(graphite) + O_2(g) \rightarrow CO_2(g) + Z\ kJ\ mol^{-1}$

At constant temperature. Then

(1) X = Y + Z

(2) –X = Y + Z

(3) X = –Y + Z

(4) X = Y – Z

Ans. (4)

65. Given below are two statements :

Statement (I): It is impossible to specify simultaneously with arbitrary precision, both the linear momentum and the position of a particle.

Statement (II) : If the uncertainty in the measurement of position and uncertainty in measurement of momentum are equal for an electron, then the uncertainty in the measurement of velocity is $\geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2m}$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

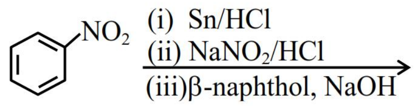
(2) Both **Statement I** and **Statement II** are true.

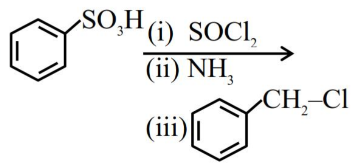
(3) **Statement I** is false but **Statement II** is true.

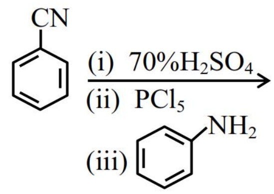
(4) Both **Statement I** and **Statement II** are false.

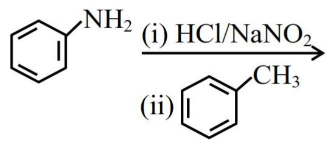
Ans. (2)

66. Which one of the following reaction sequences will give an azo dye ?

(1) 

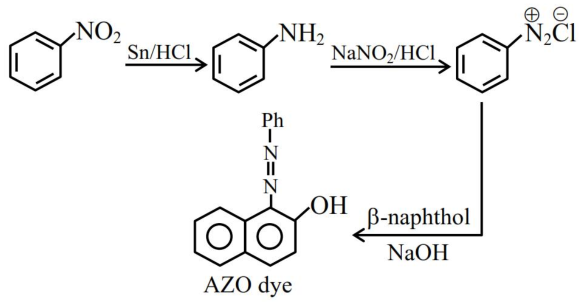
(2) 

(3) 

(4) 

Ans. (1)

Sol.



67. Drug X becomes ineffective after 50% decomposition. The original concentration of drug in a bottle was 16 mg/mL which becomes 4 mg/mL in 12 months. The expiry time of the drug in months is _____.

Assume that the decomposition of the drug follows first order kinetics.

(1) 12 (2) 2

(3) 3 (4) 6

Ans. (4)

68. The type of oxide formed by the element among Li, Na, Be, Mg, B and Al that has the least atomic radius is :

(1) A_2O_3 (2) AO_2

(3) AO (4) A_2O

Ans. (1)

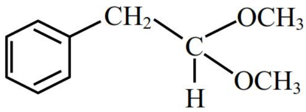
69. First ionisation enthalpy values of first four group 15 elements are given below. Choose the correct value for the element that is a main component of apatite family :

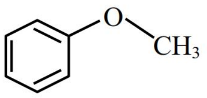
(1) 1012 kJ mol^{-1} (2) 1402 kJ mol^{-1}

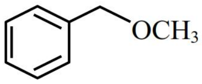
(3) 834 kJ mol^{-1} (4) 947 kJ mol^{-1}

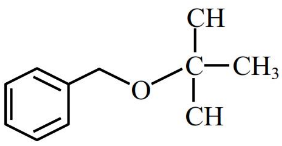
Ans. (1)

70. Which one of the following, with HBr will give a phenol?

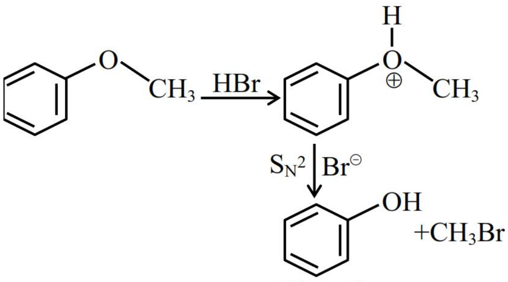
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

Phenol

SECTION-B

71. Consider the following low-spin complexes $K_3[Co(NO_2)_6]$, $K_4[Fe(CN)_6]$, $K_3[Fe(CN)_6]$, $Cu_2[Fe(CN)_6]$ and $Zn_2[Fe(CN)_6]$.

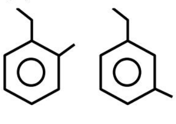
The sum of the spin-only magnetic moment values of complexes having yellow colour is _______ B.M. (answer is nearest integer)

Ans. (0)

72. Isomeric hydrocarbons $\rightarrow$ negative Baeyer's test (Molecular formula C_9H_{12})

The total number of isomers from above with four different non-aliphatic substitution sites is -

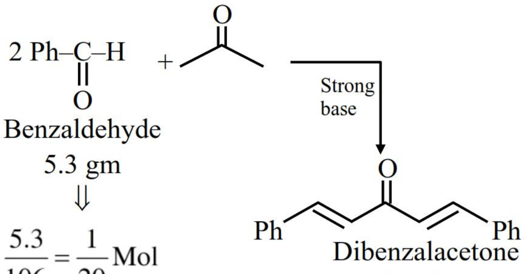
Ans. (2)

Sol. 

Above two isomers of C_9H_{12} have four different sites for aromatic electrophilic substitution reaction.

73. In the Claisen-Schmidt reaction to prepare, dibenzalacetone from 5.3 g benzaldehyde, a total of 3.51 g of product was obtained. The percentage yield in this reaction was _______ %.

Ans. (60)

Sol. 

Benzaldehyde 5.3 gm $\Downarrow$ $\frac{5.3}{106} = \frac{1}{20}$Mol

Dibenzalacetone (Product) 3.51 gm $\Downarrow$ $\frac{3.51}{234} = 0.015\,\text{Mol}$ (Actual)

Theoretical $= \frac{1}{40}$Mol

% yield $= \frac{0.015}{1/40} \times 100$

$\Rightarrow$ 60%

74. In the sulphur estimation, 0.20 g of a pure organic compound gave 0.40 g of barium sulphate. The percentage of sulphur in the compound is ____ $\times 10^{-1}$ %.

(Molar mass : O = 16, S = 32, Ba = 137 in g mol^{-1})

Ans. (275)

Sol. Organic Compound $\longrightarrow$ $BaSO_4$

0.20 gm 0.40 gm

$\frac{0.40}{233}$ mol($BaSO_4$)

$\frac{0.40}{233}$ mol (Sulphur)

$\frac{0.40}{233} \times 32$ gm (sulphur)

$$\%S = \frac{\frac{0.40 \times 32}{233} \times 100}{0.20} = 27.5\% \text{ or } 275 \times 10^{-1}\ \%$$

75. Total number of non bonded electrons present in NO_2^- ion based on Lewis theory is _____ .

Ans. (12)